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On Characterizing the Blunt Minimizes of Epsilon Invex Programs

Abstract

In this paper, we study the minimization of a differentiable epsilon invex function over an invex set and provide several new and simple characterizations of the set of all epsilon blunt minimizers of the extremum problem. This paper gives generalization of some results from Laha et al. [11]. The results of this paper extend and give approximate version of various results present in literature.

Keywords: Convexity, Non-Invex and Differentiable Functions, Epsilon Invex and Straight Functions.

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Introduction

Consider the nonlinear optimization problem

(P) min f(x) subject to $x \in$ K,

where K is a nonempty invex subset of R^n with respect to η : K $\times$K$\to R^n$ and f is a real-valued derivable function defined on an open subset D $\supseteq$ K. A vector $\bar{x} \in$ K is said to be an optional solution of the problem (P), if and only if $f(\bar{x}) \leq f(x)$ for $x \in$ K.

The characterization of the optimal solutions of the problem (P) is an important study in optimization and is useful for understanding the behavior of solution methods. Managasarian [7], Burke and Ferris [2], Penot [8], Jeyakumar et al. [4], Dinh et al. [3], Xu and Wu [9], Yang [10] and Laha et al. [0] studied various optimization problems and provided several new and simple characterizations of the solution set under convexity and generalized convexity assumptions.

In this paper, we use the notion of epsilon invexity for noninvex and differentiable functions to introduce the concept of epsilon straight functions. We present some characterizations of epsilon invex and straight functions. The characterization of the solution set of epsilon blunt minimizers of noninvex and differentiable scalar-valued epsilon invex and epsilon straight functions are obtained. The results of this paper extend and give approximate version of various results present in literature.

2. Preliminaries

In this section, we recall some known definitions and results which will be used in the sequel. The following concepts are generalizations of the concepts given in [5, 6].

Definition 2.1. A set K is said to be invex, iff for any $x, y \in$ K and $\lambda \in$ [0, 1], one has

$$x + \lambda\eta(y, x) \in K$$

Definition 2.2. Let K $\subseteq R^n$ be an invex set and let $\in > 0$ be given. A function f : K $\to$ R is said to be $\in$ - preinvex on K, iff for any $x, y \in$ K and $\lambda \in$ [0, 1], one has

$$f\big(y + \lambda\eta(x, y)\big) \leq \lambda f(x) + (1-\lambda)f(y) + \epsilon\lambda(1-\lambda)\|x - y\|.$$

If –f is $\in$-preinvex on K, then f is said to be $\in$-preincave on K. If f is both $\in$-preinvex and $\in$-preincave on K, then f is said to be $\in$-straight on K.

The following lemmas will be useful.

Lemma 2.3. Let K $\subseteq R^n$ be an invex set and let $\in > 0$ be given. Then f : K $\to$ R is differentiable $\in$ - preinvex on K, if and only if for any $x, y \in$ K one has

$$f(x) - f(y) \geq \langle \nabla f(y), \eta(x, y)\rangle - \epsilon\|x - y\|.$$

Lemma 2.4. Let K $\subseteq R^n$ be an invex set and let $\in > 0$ be given. Then f : K $\to$ R is differentiable $\in$ - preinvex on K, if and only if for any $x, y \in$ K one has

$$\langle \nabla f(y) - \nabla f(x), \eta(y, x)\rangle \geq -2\epsilon\|y - x\|.$$

Definition 2.5. (See [1]) Let $\in > 0$ be given. A vector $\bar{x} \in$ K is said to be an $\in$ - blunt minimizer of f : K $\to$ R over K, iff for any $x \in$ K one has

$$f(\bar{x}) - \epsilon\|\bar{x} - x\| \leq f(x).$$

3. Variational Inequalities

In this section, we characterize epsilon blunt minimizers of a differentiable epsilon invex function over an invex set using variational inequalities of Stampacchia and Minty type.

Theorem 3.1. Let $\in$, $\in_1 > 0$ and let $K \subseteq R^n$ be an open invex set. Let $f : K \to R$ be $\in$ - preinvex on K. Then, the following implications hold:

(i) If $\bar{x} \in K$ is an $\in_1$ – blunt minimizer of f over K, then

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -\epsilon_1 \|x - \bar{x}\|, \quad \forall x \in K;$$

(ii) If

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -\epsilon_1 \|x - \bar{x}\|, \quad \forall x \in K;$$

Then, $\bar{x}$ is an $(\in + \in_1)$ – blunt minimizer of f over K.

Proof. (i) Suppose that $\bar{x} \in K$ is an $\in_1$ – blunt minimizer of f over K.

Then, for any $x \in K$, one has

$$f(\bar{x}) - \epsilon_1 \|x - \bar{x}\| \leq f(x).$$

Since K is an invex set, for any $x \in K$ and $\lambda \in]0, 1[$, one has

$$\bar{x} + \lambda \eta(x, \bar{x}) \in K,$$

which implies that, for any $x \in K$ and $\lambda \in]0,1[$, one has

$$f(\bar{x} + \lambda \eta(x, \bar{x})) - f(\bar{x}) \geq -\in_1 \lambda \|x - \bar{x}\|.$$

Dividing throughout by λ and passing to the limits as λ tends to 0, on ehas

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -\epsilon_1 \|x - \bar{x}\|.$$

(ii) Suppose that, for any $x \in K$, one has

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -\epsilon_1 \|x - \bar{x}\|.$$

By $\in$ - invexity of f at $\bar{x}$ over K, one has

$$f(x) - f(\bar{x}) \geq \langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -\in \|x - \bar{x}\|,$$

which implies that

$$f(\bar{x}) - (\in + \in_1)\|x - \bar{x}\| \leq f(x),$$

that is, $\bar{x}$ is an $(\in + \in_1)$ – blunt minimizer of over K. This completes the proof.

Based on Theorem 3.2, for $\in = \in_1 = 0$, we have the following result.

Corollary 3.2. Let $K \subseteq R^n$ be an open invex set and let $f : K \to R$ be a preinvex function on K. Then, $\bar{x} \in K$ is an optimal solution of the problem (P) is an only if

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq 0, \forall x \in K.$$

Theorem 3.3. Let $\in$, $\in_1 > 0$ and let $K \subseteq R^n$ be an open invex set. Let $f : K \to R$ be $\in$ - preinvex on K. Then, the following implications hold:

(i) If $\bar{x} \in K$ is an $\in_1$ – blunt minimizer of f over K, then

$$\langle \nabla f(x), \eta(x, \bar{x}) \rangle \geq -(2\in + \in_1)\|x - \bar{x}\|, \quad \forall x \in K;$$

(ii) If

$$\langle \nabla f(x), \eta(x, \bar{x}) \rangle \geq -(2\in + \in_1)\|x - \bar{x}\|, \quad \forall x \in K;$$

Then, $\bar{x}$ is an $(3\in + \in_1)$ – blunt minimizer of f over K.

Proof. (i) Suppose that $\bar{x} \in K$ is an $\in_1$ – blunt minimizer of f over K.

Then, for Theorem 3.2 (i) , it follows that

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle - \epsilon_1 \|x - \bar{x}\|, \forall x \in K.$$

Since f is differentiable $\in$-preinvex on K, by Lemma 2.4, it follows that

$$\langle \nabla f(x), \eta(x, \bar{x}) \rangle - (2\in + \in_1)\|x - \bar{x}\|, \forall x \in K.$$

(ii) Suppose that,

$$\langle \nabla f(x), \eta(x, \bar{x}) \rangle - (2\in + \in_1)\|x - \bar{x}\|, \forall x \in K.$$

Since K is an open invex set, it follows that

$$\langle \nabla f(\bar{x} + \lambda(x - \bar{x})), \eta(x, \bar{x}) \rangle - (2\in + \in_1)\|x - \bar{x}\|, \forall x \in K, \forall \lambda \in (0,1).$$

Letting $\lambda \to 0^+$, it follows that

$$\langle \nabla f(\bar{x}), \eta(x, \bar{x}) \rangle \geq -(2\in + \in_1)\|x - \bar{x}\|, \forall x \in K.$$

From Theorem 3.2 (ii), it follows that, $\bar{x}$ is an $(3\in + \in_1)$ – blunt minimizer of f over K.

For $\in = \in_1 = 0$, we have the following result.

Corollary 3.4. Let $K \subseteq R^n$ be an open invex set and let $f : K \to R$ be preinvex on K. Then, $\bar{x} \in K$ is an optimal solution of the problem (P) if and only if

$$\langle \nabla f(x), \eta(x, \bar{x}) \rangle \geq 0, \forall x \in K.$$

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